

# Introduction to Simplicial Homology

Postdoc Mentor: Daniel Spiegel

Seminar Description: Homology is a pillar of algebraic topology that roughly describes how many “holes” a topological space has and the dimensions of those holes. It provides algebraic invariants that can help us differentiate between spaces. For example, if  $m$  and  $n$  are different natural numbers, then the spheres  $S^m$  and  $S^n$  are topologically distinct. The proof of this is surprisingly nontrivial but can be done using homology. More generally, the term homology refers to a certain method of associating algebraic invariants to various problems and has applications in many areas of mathematics and beyond, including in the classification of topological phases of matter. This seminar will be a gentle introduction to arguably the simplest type of homology: simplicial homology. The focus will be on understanding definitions and examples rather than proving difficult technical theorems.

*Prerequisite: MAT 150A and ideally MAT 147.*

## Sources:

- *Elements of Algebraic Topology* by James Munkres (pdf will be supplied)
- Algebraic Topology course on youtube by Norman Wildberger:  
<https://www.youtube.com/playlist?list=PL6763F57A61FE6FE8>

Format: We will meet once a week as a group for an hour (date/time flexible and based on students' availability). Before each meeting (except the first meeting), students will be expected to read a section or two of *Elements of Algebraic Topology* by Munkres (weekly reading will be no more than 10 pages) and think about an exercise. During the meeting, a student will briefly summarize the section, we'll discuss any questions, and students will attempt exercises together.

## Learning Goals:

- Work through the first chapter of *Elements of Algebraic Topology* by Munkres.
- Learn the definition of simplicial complex, abstract simplicial complex, simplicial homology, and chain complex.
- Compute the homology groups of simple examples.
- Lower the barrier for further study in algebraic topology.

## Schedule:

For the first meeting, we'll introduce ourselves and I will give a brief informal lecture to motivate the subject. Students will not need to do any reading or other preparation before the first meeting.

Aside from the first meeting, our schedule will be dictated by the sections in the first chapter of *Elements of Algebraic Topology*. We'll go through roughly one section a week, maybe two for the very short sections. See the table of contents below.

<b>Chapter 1</b>	<b>Homology Groups of a Simplicial Complex</b>	<b>1</b>
§1	Simplices	2
§2	Simplicial Complexes and Simplicial Maps	7
§3	Abstract Simplicial Complexes	15
§4	Review of Abelian Groups	20
§5	Homology Groups	26
§6	Homology Groups of Surfaces	33
§7	Zero-dimensional Homology	41
§8	The Homology of a Cone	43
§9	Relative Homology	47
*§10	Homology with Arbitrary Coefficients	51
*§11	The Computability of Homology Groups	53
§12	Homomorphisms Induced by Simplicial Maps	62
§13	Chain Complexes and Acyclic Carriers	71